# Forces Acting at a Point

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. The resultant of two forces P and Q act at right angles to P. Show that the angle between the force is $\cos^{-1}\left(-\frac{P}{Q}\right)$. Also, show that $R^2 = Q^2 - P^2$.

C2. The resultant of two force P and Q acting along the lines OA and OB respectively is at right angle of OA and the result of force P′ and Q′ acting respectively along the same straight line is at right angles to OB. Show that PP′ = QQ′.

C3. If a force F be resolved into component forces and if one component be at right angles to F and equal to $\sqrt{3}F$ in magnitude, find the direction and magnitude of the other component.

C4. A transversal cuts the lines of action of three concurrent forces P, Q, R in L, M, N respectively. If R is the resultant of P and Q, show that $\frac{P}{OL}+\frac{Q}{OM}=\frac{R}{ON}$ where O is the point of concurrence of the forces.

C5. Three coplanar forces acting on a particle are in equilibrium. The angle between the first and the second is 60° and that between the second and the third is 150°. Find the ratio of the magnitudes of the forces.

C6. ABCDEF is a regular hexagon. Forces of magnitude $4, 8\sqrt{3}, 16, 4\sqrt{3}$ and 8 Newtons acts at A in the directions AB, AC, AD, AE and AF respectively. Find the resultant of the forces.

C7. Two weights P, Q (P > Q) attached to the ends of a string rest on a smooth circular disc whose plane is vertical. Prove that the inclination θ to the horizontal of the line joining them is given by, $\tan\theta = \frac{P-Q}{P+Q}\tan\alpha,$ where 2α is a the angle subtended by PQ at the centre.

***Some Important Question***

G1. To find the magnitude and direction of the resultant of two forces acting at a point.

G2. If forces P and Q acting at an angle θ be interchanged in position, show that their resultant turns through an angle ϕ such that $\tan\frac{\phi}{2} = \frac{P-Q}{P+Q}\tan\frac{\theta}{2}$.

G3. The resultant of two forces acting at a point O in direction OA and OB and represented in magnitude by λ. OA and $\mu$. OB is represented by (λ + $\mu$). OC, where C is a point in AB such that λ. CA = $\mu$. CB.

G4. If three coplanar forces acting at a point are in equilibrium, then each is proportional to the sine of the angle between the other two.

G5. AB and AC are two strings 9m. and 12 m. long attached to pegs B and C at a horizontal distance 15 m apart. Find the tensions in the strings when a weight of 10 kg is suspended form A.

G6. Forces P–Q, P, P + Q act at a point in direction parallel to the sides of an equilateral triangle, taken in order. Find their resultant.

G7. A string ABCD is suspended from two fixed points A and D. It is carries weights of 30 kg and W kg respectively at two points B and C in it. The inclination to the vertical of AB is 30° and that of CD is 60°, the angle BCD being 120°. Find W and the tension in the different parts of the string.

Q8. A string of length 2 m is attached to two points A and B at the same level at a distance 1 m apart. A ring of 10 kg slung on the string is acted on by a horizontal force P which holds it in equilibrium vertically below B. Find the tension in the string and magnitude of P.

# Parallel Forces

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. Find the resultant of two unlike parallel forces 40 N and 5 N acting at A and B respectively where AB = 40 cm.

C2. Two like parallel forces P and (P > Q) act upon a rigid body at A and B respectively. Let P and Q be interchanged in position, then show that the point of application of the resultant will be displaced through a distance x along AB given by application of the resultant will be displacement through a distance x along AB given by $= \frac{P-Q}{P+Q}AB$.

C3. The resultant of two parallel forces whose lines of action are 12 m apart and act in opposite sense is 700 N. If the resultant acts at 3 m from one of the forces, what are the forces?

C4. If the position of the resultant of two like parallel force remains unaltered when their positions are interchanged, show that the forces are equal.

C5. Two like parallel forces P and Q act at points A and B of a body. Their resultant meets AB in C. When the forces are interchanged, the resultant meets AB in D. If AC = DC, show that P:Q ::2 : 1.

C6. P and Q are two like parallel forces. If P be moved parallel to itself through a distance x, show that their resultant moves through a distance $\frac{Px}{P+Q}$.

C7. A unitform rod 6 m long and weighing 18 kg is placed in a horizontal position upon two pegs 3 m apart. If the breaking pressure of each peg wt, find the greatest length of the portion of the rod that may project beyond either peg.

C8. Two men have to carry a block of stone weighing 300 kg by means of a light plank whose length is 6 m. How must the block be placed so that one of the men should bear a pressure of 60 kg more than the other.

***Some Important Question***

G1. A heavy uniform rod 4 m long rest horizontally on two pegs which are 1m apart. A weight of 10 kg suspended from one end or a weight of 4 kg suspended from the other end will just tilt the rod up. Find the weight of the rod and the distance of the pegs from the centre of the rod.

G2. A uniform rod of length 21 and weight W is lying across two pegs on the same level d metre apart. If neither peg can stand a stress greater than T, show that the length of the rod which can project beyond either peg cannot be greater than $l-\frac{d(W-T)}{W}$.

G3. Three like parallel forces P, Q, R act at the corners of a triangle ABC. Prove that their centre is:

(i) the centroid of the triangle if P = Q = R.

(ii) the orthocenter of the triangle if $\frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$

G4. ABC is a triangle and O is a point is the plane of the triangle. A force R acts along AO. Resolve R into two forces parallel to it and acting at B and C respectively, where

(i) O is the incentre of ΔABC

(ii) O is the circumcentre of ΔABC.

G5. The resultant of two like parallel forces P and Q passes through a point O. When P is increased by R and Q by S, the resultant still passes through O and also when Q, R replace P, Q respectively, Show that $S = \frac{Q^3}{P^2} = R - \frac{(Q-R)^2}{P-Q}$

G6. A heavy uniform rod 6 m long, rests horizontally on two pegs which are 1.5 part. A weight of 20 kg suspended from one end or a weight of 8 kg suspended from the other end will just tilt the rod up. Find the weight of the rod and the distance of the pegs from the centre of the rod.

G7. A uniform rod of weight of 50 kg and length 18 m is carried on the shoulders of two men at distance of 2 m and 3m respectively from two ends. A weight of 80 kg is also hung from the middle point of the rod. Find the total weight carried by each.

# Friction

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. To find the least force required to drag a heavy body on a rough horizontal plane.\

C2. Two equal weights are attached to the ends of a string which is laid over the top of two equally rough planes having the same altitude and placed back of back; the angles of the inclination of the planes to the horizon being 30° and 60° respectively. Show that the weights will be on the point of motion if the co-efficient of friction be $2-\sqrt{3}$.

C3. Find how high can a particle rest inside a hollow sphere of radius 'r' if the coefficient of friction be $1/\sqrt{3}$.

C4. A body of weight 80 kg rests on a rough horizontal plane while a force of 20 kg is acting on it in a direction making an angle of 60° with the horizontal. Find the force of friction that is called into play.

C5. One end of a heavy uniform rod AB can slide along a rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. If ABC be a right-angle when the rod is on the point of sliding, μ the co-efficient of friction and α the angle between AB and the vertical, show that $\mu = \frac{\tan\alpha}{2+\tan^2\alpha}$.

C6. A uniform rod rests with one extremity against a rough vertical wall, the other being supported by a string of equal length fastened to point in the wall. Prove that the least angle which the string can make with the wall is $\tan^{-1} 3/\mu$.

C7. A beam rests with one end A on a rough horizontal plane and other end against a smooth vertical wall. If $l$ be the length of the beam and α the distance of its CG from A, show that the inclination of the beam wall when on the point of slipping is $\tan^{-1}\left(\frac{l\,\mu}{a}\right)$, where μ is the coefficient of friction.

***Some Important Question***

G1. A heavy body is placed on a rough inclined plane of inclination 'α' greater than the angle of friction, being acted upon by a force parallel to the plane and along a line of greatest slope. To find the limits between which the force must lie.

G2. A weight can be just supported on a rough inclined plane by a force P acting along the plane or by a force Q acting horizontally; show that the weight is $\frac{PQ}{\sqrt{Q^2\sec^2\phi - P^2}}$, where ϕ is the angle of friction.

G3. The force acting parallel to a rough inclined plane of inclination α to the horizon, just sufficient to draw a weight up the plane is *n* times times the force which will just let it be on the point of sliding down the plane. Prove that: $\tan\alpha = \mu\frac{n+1}{n-1}$.

G4. Two equally rough bodies of weights $W_1$ and $W_2$ ($W_2 < W_1$) on a rough inclined plane are connected by a string which passes round a fixed smooth pulley in the plane. Find the greatest inclination of the plane consistent with the equilibrium of two bodies.

G5. A ladder whose centre of gravity divides it into two portions of lengths '*a*' and '*b*' rests with one end on a rough horizontal floor and the other end against a rough vertical wall. If the co-efficient of friction at the floor and wall be respectively μ and μ′, show that the inclination of the ladder to the floor, when the equilibrium is limiting, is $\tan^{-1}\frac{a-b\mu\mu}{u(a+b)}$.

G6. A uniform rod rests in a vertical plane within a fixed hemispherical bowl whose radius is equal to the length of the rod. If μ be the co-efficient of friction between the rod and the bowl, show that in limiting equilibrium the inclination of the rod to the horizontal is $\tan^{-1}\left(\frac{4\mu}{3-\mu^2}\right)$.

G7. A uniform rod rests in limiting equilibrium within a rough vertical circle. If the rod subtends and angle 2α at the centre of the circle and if λ be the angle of friction, show that the angle of inclination of the rod to the vertical is $\tan^{-1}\left(\frac{\cos 2\alpha + \cos 2\lambda}{\sin 2\lambda}\right)$.

G8. A ladder inclined at 60° to the horizon rests between a rough floor and a smooth vertical wall. Show that if the ladder begins to slide down when a man has ascended so that his centre of gravity is half way up, coefficient of friction between the foot of ladder and the floor is $\frac{\sqrt{3}}{6}$.

# *Centre of Gravity*

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. Find centre of gravity of a thin uniform triangular lamina.

C2. A square hole is punched out of a circular lamina, the diagonal of the square, being a radius of the circular lamina. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8\pi - 4}$ from the centre of the circle, where a is the diameter of the circle.

C3. A uniform circular lamina of radius 3*a* and centre O has a hole in the form of an equilateral triangle of side 2*a* with one vertex at O. Prove that the distance of the C.G. from O is $\frac{2a}{9\pi - \sqrt{3}}$.

C4. Find the centre of gravity of a right circular solid cone.

C5. Find the centroid (C.G.) of a plane lamina in form of a quadrant of an ellipse when matter is distributed uniformly.

C6. Find the centre of gravity of the area of position of the curve $y^2(2a - x) = x^3$ and its asymptote whose equation is $x = 2a$.

C7. $y = \sin x$ between $x = 0$ and $x = \pi$.

***Some Important Question***

G1. Find centre of gravity of a lamina in the form of a trapezium.

G2. A uniform triangular lamina ABC is suspended from the vertex A. Prove that the inclination ofhte side BC to the horizontal is $\tan^{-1}\frac{b^2 - c^2}{4\Delta}$, where $\Delta$ is the area of the triangle ABC.

G3. A uniform lamina in the form of a right-angled triangle is suspended by a string attached to the right angle. If the sides containing the right angle are in the ratio 3 : 1, prove that in the position of equilibrium the hypotenuse is inclined at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ to the vertical.

G4. Find the centre of gravity of a segment of a circle.

G5. Find the centre of grvity of the arc of the asteroid $^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant.

G6. Find the centre of gravity of the area of the curve $x^{2/3} + v^{2/3} = a^{2/3}$ lying in the 1st quadrant.

G7. Find the centre of gravity of the arc of the cardiod $= a(1 + \cos\theta)$ lying above the initial line.

G8. Find the position the centroids of the areas enclosed by curves $y^2 = ax$ and $x^2 = by$.

# Virtual Work

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. The virtual works done by the tension in a virtual extension of a string from length $l$ to $l+\delta l$ is T $\delta l$, where T is the tension in the string.

C2. The virtual works done by the thrust in a virtual extension of a light rod from length $l$ to $l + \delta l$ is T. $\delta l$, where T is the thrust in the rod.

C3. Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Prove that its tension is 3 *W*.

C4. Four equal jointed rods each of length *a*, are hung from an angular point which is connected by an elastic string with the opposite point. If the rods hang in the form of a square and if the modulus of elasticity of the string be equal to the weight of *a* rod, show that the natural length of the string is $\frac{a\sqrt{2}}{3}$.

C5. Four equal uniform rods each of weight *w* are freely joined to form a rhombus ABCD. The frame work is suspended freely from A and a weight W is attached to each of the joints B, C and D. If two horizontal forces each of the magnitude P acting at B and D keep the angle BAD equal to 120°, prove that P = (W + *w*) $2\sqrt{3}$.

C6. Four equal uniform rods, each of weight W are joined so as to form a square ABCD, the side AB is fixed in vertical position with A uppermost and the figure is kept in shape by a string joining the middle points of AD and DC. Find the tension of the string.

C7. A uniform beam of length 2*a* rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium, the rod is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

***Some Important Question***

G1. Principal of Virtual Work for a System of Coplanar Forces Acting on a Particle (Vector Method).

Statement: The necessary and sufficient condition that a particle acted upon by a number of coplanar forces be in equilibrium, is that the sum of the virtual works done by the forces in any small virtual displacement, consistent with the geometrical conditions of the system, is zero.

G2. Two equal uniform rods AB and AC, each of length 2*b* are freely joined at A and rest on a smooth vertical circle of radius *a*. Show that if 2θ be the angle between them, then $b\sin^3\theta = a\cos\theta$.

G3. Six equal heavy rods, freely hinged at the ends, form a regular hexagon ABCDEF, which hung up by the corner A is kept from altering its shape by two light rods BF and CE. Prove that the thrust in these rods are $\frac{1}{2}5\sqrt{3}W$ and $\frac{1}{2}\sqrt{3}W$, where W is the weight of each rod.

G4. A heavy uniform rod of length 2$a$ rests with its ends in contact with two smooth inclined planes of inclination α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principle of virtual work that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$

G5. A heavy elastic string, whose natural length 2πa is placed round a smooth cone whose axis is vertical and whose semi-vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is $\alpha\left[1+\frac{W\cot\alpha}{2\pi\lambda}\right]$.

G6. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth v vertical wall with which the curved surface is in contract. If θ and ϕ, are the inclinations of the string and the plane base of the hemisphere to the vertical, show that $\tan\phi = \frac{3}{8} + \tan\theta$.

G7. Five equal uniform rods, freely jointed at their ends form a regular pentagon ABCDE and BE is joined by a light rod. The system is suspended from A in the vertical plane. Prove that the thrust in BE is $W\cot\left(\frac{\pi}{10}\right)$, where W is the weight of each rod.

G8. A frame consists of five bars forming the sides of a rhombus ABCD with diagonal AC. If four equal forces P act inwards at the middle points of the sides and at right angle to the respective sides, prove that the tension in AC is $\frac{P\cos 2\theta}{\sin\theta}$, where θ denote the angle BAC.

# *Forces in Three Dimensions*

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. To show that the quantities (LX + MY + NZ) and ($X^2$ + $Y^2$ + $Z^2$) are invariants for any given system of forces, whatever origin and axes may be chosen.

C2. To find the equation of the central of any given system of forces acting on a rigid body.

C3. There forces P, Q and R act along three non-intersecting edges of a cube; find the central axis.

C4. Two forces P and Q act along the straight lines whose equations are y = x, z = c and y = –x, z = –c respectively. Show that their central axis lies on the straight line. $y = x\frac{P-Q}{P+Q}$ and $\frac{z}{c} = \frac{P^2 - Q^2}{P^2 + Q^2}$.

C5. A single force is equivalent to components forces X, Y, Z along the axes of co-ordinates and to couple L, M, N about these axes. Prove that the magnitude of the single force is $\sqrt{X^2 + Y^2 + Z^2}$ and the equations to its lines of action are $\frac{yZ - zY}{L} = \frac{zX - xZ}{M} = \frac{xY - yX}{N} = 1$.

C6. Two forces act, one along the line y = 0, z = 0 and the other along the line x = 0, z = c. As the forces vary, show that the surface generated by the axis of their equivalent wrench is $(x^2 + y^2)z = cy^2$.

C7. Three forces each of magnitude P and acting in the positive direction of the axes have their lines of action $-y = z = a;\ -z = x = a; -x = y = a.$ Prove that they are equivalent to a force P $\sqrt{3}$ at the origin and a couple.

***Some Important Question***

G1. Any system of forces acting an a rigid body can be reduced in general to a force acting at an arbitrary chosen point of the body and a couple.

G2. To show that every given system of forces acting on a rigid body can be reduced to a wrench.

G3. Equal forces act along two perpendicular diagonals of opposite faces of a cube of side a. Show that they are equivalent tot single force R acting along a line through the centre of the cube and a couple 1/2aR with the same line for axis.

G4. A force *P* acts along the axis of x and another force *nP* along a generator of the cylinder $x^2 + y^2 + a^2$. Show that the central axis lies on the cylinder $n^2(nx - z)^2 + (1 + n^2)^2 y^2 n^4 a^2$.

G5. Two forces P and Q act along the straight lines whose equatinos are $y = x\tan\alpha, z = c$ and $y = -x\tan\alpha, x = -c$ respectively. Show that their central axis lies on a straight line

$$\frac{y}{x}=\frac{P-Q}{P+Q}\tan\alpha \quad \text{and} \quad \frac{z}{c}=\frac{P^2-Q^2}{P^2+2PQ\cos 2\alpha+Q^2}$$

For all values of P and Q, prove that this line is a generator of the surface $(x^2+y^2)z\sin 2\alpha = 2cxy$.

G6. A force F acts along the axis and a force nF along a straight line, intersecting the axis of y at a distance c from the origin and parallel to the plane of *zx*. Show that as this straight line turns round the axis of y, the central axis of the forces generates the surface $[x^2x^2+(n^2-1)z^2](c-y)^2 = y^2x^2$.

G7. A force parallel to the axis of z acts at the point ($\alpha$, 0, 0) and an equal force perpendicular to the axis of z acts at the point (–a, 0, 0). Show that the central axis of the system lies on the surface $z^2(x^2+y^2)=(x^2+y^2-ax)^2$.

G8. A force P acts along the axis of x and another force 3P along a generator of the cylinder $x^2+y^2=a^2$. Show that the central axis lies on the cylinder $9(3x-z)^2+100y^2=81a^2$.

# *Wrenches, Null Lines and Null Planes,*

**Time : 1 Hour** **Maximum Marks: 40**

***Compulsory Questions***

C1. If P and Q be two non-intersecting forces whose directions are perpendicular, show that the ratio of distance of the central axis their lines of action are $Q^2$ to $P^2$.

C2. Two forces P and Q are such that their central axis is given in position and the line of action of P is given. Show that the locus of the line of action of Q is a conicoid.

C3. To find the equation to the null plane of a given point (a, b, c) referred to any axes Ox, Oy, Oz.

C4. To find the condition that the straight line $\frac{x-f}{l}=\frac{y-g}{n}=\frac{z-h}{n}$ may be a null line for the system of forces (X, Y, Z; L, M, N)

C5. To find the equation of the conjugate line of the given line $\frac{x-f}{l}=\frac{y-g}{m}=\frac{z-h}{n}$.

C6. Find the null point of the plane $x+y+z=0$ for the force system (X, Y, Z ; L, M, N)

C7. Show that among that null lines of any system of forces, four are generators of any hyperboloid, two belonging to one system of generators and two to the other.

***Some Important Question***

G1. To show that a given system of forces can be replaced by two forces equivalent to the given system, in an infinite number of ways and that the tetrahedron formed by the two forces is of constant volume.

G2. The axes of two given wrenches interest at right angles. Their intensities are X and Y and their pitches are $p_x$ and $p_y$. If the pitches are given, to find the locus of the central axis.

G3. Any wrench may be resolved into two wrenches, whose axes intersect at right angles, in an infinite number of ways.

G4. Show that the minimum distance between two forces which are equivalent to a given system (R, K) and which are inclined at a given angle $2\alpha$ is $\frac{2K}{R}\cot\alpha$ and the forces are then each equal to $\left(\frac{R}{2}\right)\sec\alpha$.

G5. Wrenches of the same pitch p act along the edges of a regular tetrahedron ABCD of side a. If the intensities of the wrenches along AB, DC are the same and also those along BC, DA and DB, CA; show that the pitch of the equivalent wrench is $\left(P+\frac{a}{2\sqrt{2}}\right)$.

G6. To find the null point of the plane $lx + my + nz = 1$ for the system of forces (X, Y, Z; L, M, N)

G7. A system of forces given by (X, Y, Z; L, M, N) is replaced by two forces, one acting along the axis of x and another force. Show that the magnitude of the forces are

$\frac{LX + MY + NZ}{L}$ *and* $\frac{\left[(MY + NZ)^2 L^2 \left(Y^2 + Z^2\right)\right]^{1/2}}{L}$ and also find the equation of the line of action of the other force.

G8. Show that the wrench (X, Y, Z; L, M, N) is equivalent to the forces, one along the line x = y = z and the other along the line given by

$$Lx + My + Nz = 0, x(Y - Z) + y(Z - X) + Z(X - Y) = L + M + N$$

and find the magnitude of the two forces.